INTERNAL ENERGY IN DISSIPATIVE RELATIVISTIC FLUIDS

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ABSTRACT. Liu procedure is applied to a first order weakly nonlocal special relativistic fluid. It is shown, that a reasonable relativistic theory is and extended one, where the basic state space contains the momentum density. This property follows from the structure of the energy-momentum balance and the Second Law of thermodynamics. Moreover, the entropy depends on the energy density and the momentum density on a given specific way, indicating that the local rest frame energy density cannot be interpreted as the internal energy, the local rest frame momentum density should be considered, too. The corresponding constitutive relations for the stress and the energy flux are derived.

1. INTRODUCTION

Nonrelativistic nonequilibrium thermodynamics separates the dissipative and nondissipative parts of the evolution of physical quantities. This separation is based on the construction of the internal energy balance [1, 2, 3]. According to the classical interpretation the internal energy is the difference of the total energy and the known specific energy types. The entropy function depends directly on the internal energy. The internal energy is distributed unbiased among the molecular degrees of freedom. The process how the other energy types are converted to internal energy is called dissipation. This approach is common in every theories of nonequilibrium thermodynamics including classical irreversible thermodynamics, where the hypothesis of local equilibrium applies. However, there is nothing similar in relativistic irreversible thermodynamics. Furthermore, practically there is no relativistic irreversible thermodynamics at all, because the local equilibrium theory is plagued by serious controversies, therefore only the extended theories, theories beyond local equilibrium, are considered as viable. The reason is that the classical theory of Eckart for relativistic fluids [4] is simple and elegant, but produces generic instabilities [5]. The more developed extended theories incorporate the theory of Eckart, therefore inherit (but more or less suppress) the instabilities [6, 7, 8].

In this paper we investigate the possibility of local equilibrium in relativistic hydrodynamics by methods of continuum thermodynamics. At the next section the balances of energy-momentum and entropy are introduced. In the third section we calculate the dissipation inequality for local equilibrium (first order) relativistic hydrodynamics by Liu procedure. The need of second order (extended, or weakly nonlocal) theories is indicated by the emergent structure. A new concept of relativistic internal energy follows. Based on these results we give the constitutive equations of the simplest extended theory by the heuristic arguments of irreversible thermodynamics in the fourth section.

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2. Basic balances of relativistic fluids

For the metric (Lorentz form) we use the $g^{\mu\nu} = diag(-1, 1, 1, 1)$ convention and the units are introduced that the speed of light $c = 1$. Therefore for a four-velocity u^α we have $u_\alpha u^\alpha = -1$. $\Delta^\alpha{}_\beta = g^\alpha{}_\beta + u^\alpha u_\beta$ denotes the u-orthogonal projection. First we give the basic balances of energy-momentum and entropy.

The energy-momentum density tensor is given with the help of the rest-frame quantities

$$T^{\alpha\beta} = eu^\alpha u^\beta + u^\alpha q^\beta + u^\beta q^\alpha + P^{\alpha\beta}, \tag{1}$$

where $e = u_\alpha u_\beta T^{\alpha\beta}$ is the *density of the energy*, $q^\beta = -u_\alpha \Delta^\beta{}_\gamma T^{\alpha\gamma}$ is the *energy flux* or *heat flow* $q^\alpha = -u_\beta \Delta^\alpha{}_\gamma T^{\gamma\beta}$ is the *momentum density* and $P^{\alpha\beta} = \Delta^\alpha_\gamma \Delta^\beta_\mu T^{\gamma\mu}$ is the *pressure tensor*. The momentum density, the energy flux and the pressure are spacelike in the comoving frame, therefore $u_\alpha q^\alpha = 0$ and $u_\alpha q^\alpha = 0$ and $u_\alpha P^{\alpha\beta} = u_\alpha P^{\beta\alpha} = 0^\beta$. The energy-momentum tensor is symmetric, because we assume that the internal spin of the material is zero. In this case the energy flux and the momentum density are equal. Let us emphasize that the form (1) of the symmetric energy-momentum tensor is completely general for one-component fluids, nevertheless it is expressed by the local rest frame quantities.

Now the conservation of energy-momentum $\partial_\beta T^{\alpha\beta} = 0$ is expanded to

$$\partial_\beta T^{\alpha\beta} = \dot{e}u^\alpha + eu^\alpha \partial_\beta u^\beta + e\dot{u}^\alpha + u^\alpha \partial_\beta q^\beta + q^\beta \partial_\beta u^\alpha + \dot{q}^\alpha + q^\alpha \partial_\beta u^\beta + \partial_\beta P^{\alpha\beta}, \tag{2}$$

where $\dot{e} = \frac{d}{d\tau}e = u^\alpha \partial_\alpha e$ denotes the derivative of e by the proper time τ. Its timelike part in a local rest frame gives the balance of the energy

$$-u_\alpha \partial_\beta T^{\alpha\beta} = \dot{e} + e\partial_\alpha u^\alpha + \partial_\alpha q^\alpha + q^\alpha \dot{u}_\alpha + P^{\alpha\beta}\partial_\beta u_\alpha = 0. \tag{3}$$

The spacelike part in the local rest frame describes the balance of the momentum

$$\Delta^\alpha{}_\gamma \partial_\beta T^{\gamma\beta} = e\dot{u}^\alpha + q^\alpha \partial_\beta u^\beta + q^\beta \partial_\beta u^\alpha + \Delta^\alpha{}_\gamma \dot{q}^\gamma + \Delta^\alpha{}_\gamma \partial_\beta P^{\gamma\beta} = 0^\alpha. \tag{4}$$

The entropy density and flux can also be combined into a four-vector, using local rest frame quantities:

$$S^\alpha = su^\alpha + J^\alpha, \tag{5}$$

where $s = -u_\alpha S^\alpha$ is the *entropy density* and $J^\alpha = S^\alpha - u^\alpha s = \Delta^\alpha{}_\beta S^\beta$ is the *entropy flux*. The entropy flux is u-spacelike, therefore $u_\alpha J^\alpha = 0$. Now the Second Law of thermodynamics is expressed by the following inequality

$$\partial_\alpha S^\alpha = \dot{s} + s\partial_\alpha u^\alpha + \partial_\alpha J^\alpha \geq 0. \tag{6}$$

3. Thermodynamics

The thermodynamical background in relativistic theories is usually based on analogies with nonrelativistic thermostatics. However, nonequilibrium thermodynamics developed beyond the simple 'let us substitute everything into the entropy balance and see what happens' theory since Eckart. It is important to check the dynamic consistency of the Second Law, considering the evolution equations as constraints for the entropy balance. This method of nonequilibrium thermodynamics is constructive, gives important information for new theories and reveals some deeper interrelations. Here we exploit the Second Law by Liu's procedure [9] introducing a first order weakly nonlocal state space in all basic variables, thus restricting ourselves to a local equilibrium theory. One can find a general treatment of nonrelativistic classical and extended irreversible thermodynamics from this point of view

in [10]. Our aim here is to investigate the relativistic fluids with similar methods to get the relativistic equivalent of the classical Fourier-Navier-Stokes system of equations for one component fluids.

Our most important assumption regarding relativistic thermodynamics is that the constitutive equations are local rest frame expressions. From a physical point of view it is natural, because material interactions are local.

The *basic state space* of the theory is spanned by the energy density e and by the velocity field u^α. The *constitutive state space* is spanned by the basic state variables and their first derivatives, therefore it is first order weakly nonlocal. Hence the constitutive functions depend on the variable set $C = (e, u_\alpha, \partial_\alpha e, \partial_\alpha u_\beta)$. The *constitutive functions* are the energy flux/momentum density q^α, the pressure $P^{\alpha\beta}$, the entropy density s and the entropy flux J^α. The derivatives of the constitutive functions are denoted by the serial number of the corresponding variable in the constitutive space, e.g. $\frac{\partial s}{\partial(\partial_\alpha e)} = \partial_3 s$. With this notation we can distinguish easily the derivatives by the constitutive and spacetime variables. A nonequilibrium thermodynamic theory is considered to be solved if all other constitutive quantities are expressed by the entropy density and its derivatives.

According to the procedure of Liu the balance of energy-momentum (2) is a constraint to the entropy balance (6) with the Lagrange-Farkas multiplier Λ_α

$$\partial_\alpha S^\alpha - \Lambda_\alpha \partial_\beta T^{\alpha\beta} \geq 0. \tag{7}$$

Let us remember, that here the spacelike components of the four quantities and the entropy density are the constitutive quantities depending on the introduced constitutive variables C. Therefore, in the above inequality we can develop the derivatives of the composite functions. The coefficients of the derivatives that are not in the constitutive space must be zero, therefore we get the following Liu-equations:

$$\partial_{\alpha\beta} e \quad : \quad (\partial_3 S^\alpha)^\beta - \Lambda_\mu (\partial_3 T^{\mu\alpha})^\beta = 0^{\alpha\beta}, \tag{8}$$

$$\partial_{\alpha\beta} u_\gamma \quad : \quad (\partial_4 S^\alpha)^{\beta\gamma} - \Lambda_\mu (\partial_4 T^{\mu\alpha})^{\beta\gamma} = 0^{\alpha\beta\gamma}. \tag{9}$$

The simple structure of the Liu equations suggests the assumption that the Lagrange multiplier is a local function, it does not depend on the derivatives of the basic state variables

$$\Lambda_\gamma = \Lambda_\gamma(n, e). \tag{10}$$

Then a general solution of (8)-(9) is

$$S^\alpha - \Lambda_\gamma T^{\gamma\alpha} - A^\alpha = 0^\alpha, \tag{11}$$

where $A^\alpha = A^\alpha(n, e)$ is an arbitrary local function.

Let us introduce the splitting of the vector multiplier and the four-vector A^α to spacelike and timelike parts in the local rest frame as

$$\begin{aligned} \Lambda^\alpha &= -\Lambda u^\alpha + l^\alpha, \\ A^\alpha &= A u^\alpha + a^\alpha, \end{aligned}$$

where for the spacelike components $u_\alpha l^\alpha = u_\alpha a^\alpha = 0$. Now equation (11) gives

$$u^\alpha(s - \Lambda e - l_\gamma q^\gamma - A) + (J^\alpha - \Lambda q^\alpha - l_\gamma P^{\gamma\alpha} - a^\alpha) = 0^\alpha. \tag{12}$$

Here both the timelike and spacelike parts are zero, resulting in

$$
\begin{aligned}
s &= \Lambda e + l_\gamma q^\gamma + A, && (13)\\
J^\alpha &= \Lambda q^\alpha + l_\gamma P^{\gamma\alpha} + a^\alpha. && (14)
\end{aligned}
$$

After the identification of the Liu equations we can get the *dissipation inequality* as

$$
\begin{gathered}
\partial_\alpha e\left[(\partial_1 s)u^\alpha + \partial_1 J^\alpha - \Lambda u^\alpha - \Lambda\partial_1 q^\alpha - l_\gamma \partial_1 P^{\gamma\alpha} - l_\gamma \partial_1 q^\gamma u^\alpha\right] + \\
\partial_\alpha u_\beta \left[(s - \Lambda e - l_\gamma q^\gamma)\,\Delta^{\alpha\beta} + (\partial_2 s)^\beta u^\alpha + (\partial_2 J^\alpha)^\beta - \right.\\
\left. l^\beta e u^\alpha - l^\beta q^\alpha - \Lambda(\partial_2 q^\beta)^\alpha - \Lambda_\gamma(\partial_2 P^{\gamma\alpha})^\beta - \Lambda_\gamma u^\alpha(\partial_2 q^\gamma)^\beta\right] \geq 0. \qquad (15)
\end{gathered}
$$

Here we exploited that the partial differentiation by e can be exchanged with the multiplication by the four velocity u^α.

In the dissipation inequality one should consider the solution of the Liu-equations. Substituting (13) and (14) into (15) we get

$$
\begin{gathered}
\partial_\alpha e\left[(\partial_1 s - \Lambda - l_\gamma\partial_1 q^\gamma)u^\alpha + q^\alpha\partial_1\Lambda + P^{\gamma\alpha}\partial_1 l_\gamma + \partial_1 a^\alpha\right] + \\
\partial_\alpha u_\beta\left[A\Delta^{\alpha\beta} + q^\alpha(\partial_2\Lambda)^\beta + P^{\gamma\alpha}(\partial_2 l_\gamma)^\beta + (\partial_2 a^\alpha)^\beta + \right.\\
\left. u^\alpha\left((\partial_2 s)^\beta - l_\gamma(\partial_2 q^\gamma)^\beta - l^\beta e - \Lambda q^\beta\right) - l^\beta q^\alpha - \Lambda P^{\alpha\beta}\right] \geq 0. \qquad (16)
\end{gathered}
$$

Here the following identities were applied simplifying the last term ($\partial_2 = \partial_{u_\beta}$)

$$
\begin{aligned}
u_\gamma \partial_{u_\beta} q^\gamma &= \partial_{u_\beta}(u_\gamma q^\gamma) - q^\gamma\partial_{u_\beta}u_\gamma = -q^\gamma {\Delta_\gamma}^\beta = -q^\beta,\\
u_\gamma \partial_{u_\beta} P^{\gamma\alpha} &= \partial_{u_\beta}(u_\gamma P^{\gamma\alpha}) - P^{\gamma\alpha}\partial_{u_\beta}u_\gamma = -P^{\gamma\alpha}{\Delta_\gamma}^\beta = -P^{\beta\alpha}.
\end{aligned}
$$

Observing the first term in the last form of the dissipation inequality one can eliminate the direct velocity dependence of the entropy function recognizing that the entropy may depend on the energy flux in the following form

$$
s(e, u^\alpha, \partial_\alpha e, \partial_\alpha u^\beta) = \hat{s}(e, q^\gamma(e, u^\alpha, \partial_\alpha e, \partial_\alpha u^\beta)). \qquad (17)
$$

Therefore the entropy is local, independent of the derivatives of the basic state space variables and the velocity field, but depends on the energy flux. The energy flux can depend also on the derivatives because according to our initial assumptions it is considered as a constitutive function. Then the Lagrange-Farkas multipliers are determined by the entropy derivatives

$$
\partial_e\hat{s} = \Lambda, \quad \partial_{q^\alpha}\hat{s} = l_\alpha. \qquad (18)
$$

We introduce a temperature T as

$$
\partial_e\hat{s} = \Lambda = \frac{1}{T}. \qquad (19)
$$

We may recognize that a full thermostatic compatibility requires that in (13): $A := \frac{p}{T}$, where p is the pressure. These consequences are completely analogous to the results of the nonrelativistic nonequilibrium thermodynamic theory, where thermostatics arises from the structure of the balance form evolution equations as constraints for the Second Law.

Finally we assume that entropy flux is classical and the additional term a^α [11] is zero

$$
a^\alpha = 0^\alpha. \qquad (20)
$$

Then the dissipation inequality reduces to the following simple form

$$q^\alpha\partial_\alpha\frac{1}{T} - \frac{1}{T}\left(P^{\alpha\beta} + Tl^\beta q^\alpha - p\Delta^{\alpha\beta}\right)\partial_\alpha u_\beta - P^{\alpha\gamma}\partial_\alpha l_\gamma - \left(\frac{q^\alpha}{T} + el^\alpha\right)\dot{u}_\alpha \geq 0. \tag{21}$$

As we do not want an acceleration dependent entropy production, we must require that the last term vanishes. According to (18) and (19)

$$e\partial_{q_\alpha}\hat{s} + q^\alpha\partial_e\hat{s} = 0. \tag{22}$$

The general solution of (22) can be given as

$$\hat{s} = \tilde{s}(e^2 - q^\alpha q_\alpha) + B, \tag{23}$$

where B=const. The entropy must depend on the energy density e and the momentum density q^α on a very particular but simple way. As a consequence of this functional form of the entropy function the Gibbs relation can be given with the help of the entropy derivatives (18) as

$$de - \frac{q^\alpha}{e}dq_\alpha = Tds. \tag{24}$$

We may require first order homogeneity of the entropy in (23), without restricting the generality. That can we get introducing $E = \sqrt{|e^2 - q_\alpha q^\alpha|}$ as a variable of the entropy density. In this way the entropy is a first order homogeneous functions both of the energy density e and the momentum density q^α. With this property it is unique.

The corresponding potential relation can be constructed according to the first order homogeneity of the physical quantities as

$$e - \frac{q^\alpha q_\alpha}{e} = Ts - p. \tag{25}$$

The previous thermostatic relations require the interpretation of E as internal energy. On the other hand let us recognize that E is the absolute value of the energy vector

$$E = \|E^\alpha\| = \| - u_\beta T^{\beta\alpha}\| = \|eu^\alpha + q^\alpha\| = \sqrt{|e^2 - q_\alpha q^\alpha|}. \tag{26}$$

However, one should pay attention that the $1/T$ introduced in (19) is not the derivative of the entropy function according to E.

Finally the entropy flux from (14) and (20)

$$J^\alpha = \frac{1}{T}q^\alpha - \frac{q_\gamma}{eT}P^{\gamma\alpha}. \tag{27}$$

The final form of the dissipation inequality is

$$q^\alpha\partial_\alpha\frac{1}{T} - \frac{1}{T}\left(P^{\alpha\beta} + \frac{q^\beta q^\alpha}{e} - p\Delta^{\alpha\beta}\right)\partial_\alpha u_\beta - P^{\alpha\gamma}\partial_\alpha\frac{q_\gamma}{Te} \geq 0. \tag{28}$$

The last term in this expression with a derivative of one of the constitutive quantities indicates that we cannot give proper thermodynamic fluxes and forces, as a solution of the inequality. An other problem appeared already with (22), because l_α, as the spacelike part of the Lagrange multiplier in a local rest frame, was assumed independent of the derivatives of e and u^α. That is the Fourier heat conduction was excluded as a possible constitutive function. Both problems indicate that a complete theory may exist only either in an enlarged constitutive space or in an extended basic state space. One of the possibilities is to introduce higher order derivatives of the basic state space into the constitutive state space and construct

a second order weakly nonlocal theory. The other possibility is to enlarge the basic state space and construct an extended theory. In both cases the key that may lead beyond the traditional Müller-Israel-Stewart theory is the new internal energy E.

4. Extended irreversible thermodynamics of relativistic fluids

Motivated by the results of the previous section we calculate the entropy production by a direct substitution of the balance of the energy into the entropy balance. We are to construct an extended theory, introducing q^α as an independent variable, but exploiting the fact that the entropy depends both on the energy and momentum densities by the specific way derived above.

The entropy flux is assumed to have the very classical form

$$J^\alpha = \frac{1}{T} q^\alpha. \tag{29}$$

Substituting the energy balance (3) into the entropy balance equation we arrive at the following entropy production formula:

$$\begin{aligned} &\partial_\alpha S^\alpha = \dot{s}(e^2 + q^\alpha q_\alpha, s) + s\partial_\alpha u^\alpha + \partial_\alpha J^\alpha = \\ &-\frac{1}{T}(e\partial_\alpha u^\alpha + \partial_\alpha q^\alpha + q^\alpha \dot{u}_\alpha + P^{\alpha\beta}\partial_\beta u_\alpha) + \frac{q^\alpha}{Te}\dot{q}_\alpha + s\partial_\alpha u^\alpha + \partial_\alpha \left(\frac{1}{T} q^\alpha\right) = \\ &-\frac{1}{T}\left(P^{\alpha\beta} - (-e + sT)\Delta^{\alpha\beta}\right)\partial_\alpha u_\beta + q^\alpha \left(\partial_\alpha \frac{1}{T} - \frac{\dot{u}^\alpha}{T} - \frac{\dot{q}^\alpha}{eT}\right) \geq 0 \end{aligned} \tag{30}$$

In isotropic continua the above entropy production results in constitutive functions assuming a linear relationship between the thermodynamic fluxes and forces. The thermodynamic fluxes are the *viscous stress* $\Pi^{\alpha\beta} = \left(P^{\alpha\beta} - (sT - e)\Delta^{\alpha\beta}\right)$, and the energy flux q^α. We get

$$\Pi^{\alpha\beta} = P^{\alpha\beta} - \Delta^{\alpha\beta}\left(p - \frac{q^\beta q_\beta}{e}\right) = -2\eta(\Delta^{\alpha\gamma}\Delta^{\beta\mu}\partial_\gamma u_\mu)^{s0} - \eta_v \partial_\gamma u^\gamma \Delta^{\alpha\beta}, \tag{31}$$

$$q^\alpha = -\lambda \frac{1}{T^2}\Delta^{\alpha\gamma}\left(\partial_\gamma T + T\dot{u}^\alpha + \frac{T\dot{q}^\alpha}{e}\right), \tag{32}$$

where ^{s0} denotes symmetric traceless part of the corresponding second order tensor $(A^{ij})^{s0} = \frac{1}{2}(A^{ij} + A^{ji}) - \frac{1}{3}A^{ll}\delta^{ij}$ and we have introduced the scalar thermostatic pressure according to (25) (therefore $p \neq P^\alpha_\alpha/3$). (31) and (32) are the relativistic generalizations of the Newtonian viscous stress function and the Fourier law of heat conduction. The shear and bulk viscosity coefficients, η and η_v and the heat conduction coefficient λ are nonnegative, according to the inequality of the entropy production (30).

The equations (3), (4) are the evolution equations of a relativistic heat conducting ideal fluid, together with the constitutive functions (31) and (32). As special cases we can get the relativistic Navier-Stokes equation substituting (31) into (4) and assuming $q^\alpha = 0$, or the relativistic heat conduction equation substituting (32) into (3) assuming that $\Pi^{\alpha\beta} = 0$. The heat conduction part results in a special extended theory, where only the energy flux appears as independent variable.

5. Summary and discussion

Int the first part of the paper we have investigated the local equilibrium theory of special relativistic fluids. We have seen that there may be no such theory that could give a complete solution of the entropy inequality provided the following conditions

(1) local Lagrange-Farkas multipliers,
(2) local entropy (17),
(3) no additional term in the entropy flux (20).

The first two assumptions were necessary to get a particular solution of the Liu equations and the dissipation inequality. On the other hand they are natural in local equilibrium.

We have concluded that either an extension of the basic state space or an enlargement of the constitutive state space can give a complete solution. Our investigations indicated a particular dependence of the entropy on the energy and momentum densities, leading to a distinction of internal and total energy densities of relativistic fluids.

The local rest frame energy density $e = u_\alpha T^{\alpha\beta} u_\beta$ is usually interpreted as internal energy in thermodynamic theories. However, the symmetry of the energy-momentum tensor can hide that energy flux is related to dissipation, but momentum density is not. According to the previous investigations the total energy density e - the time-timelike part of the energy-momentum tensor - is not a suitable internal energy, the entropy density should be a function of the absolute value of the energy vector $E^\alpha = -u_\beta T^{\alpha\beta}$ - the timelike part of the energy momentum.

To compare our proposal to the traditional Müller-Israel-Stewart theory [12, 13] it is instructive to expand the internal energy into series, assuming that $e^2 > q^\alpha q_\alpha$

$$E = \sqrt{|e^2 - q^\alpha q_\alpha|} \approx e - \frac{\mathbf{q}^2}{2e} + ... \tag{33}$$

The last, quadratic term in the above expression is what appears in the Müller-Israel-Stewart theory. However, in our case

– the corresponding relaxation time is fixed $\tau = 1/e$,
– the quadratic term is only the first approximation,
– only the energy flux was introduced as an independent variable in our extended theory, the viscous stress was not necessary.

The series expansion is instructive comparing to nonrelativistic hydrodynamics. There the internal energy is the difference of the total energy and the relative kinetic energy. In (33) the quadratic expression is what one could consider as a kind of energy of the flow, but of course it is not connected to an external observer, it considers only the local rest frame momentum density. In a sense our expression shows that introducing E as internal energy we declared that the energy of the flow (in the local rest frame) does not give a dissipative contribution.

The extension of the present calculations considering the balance of particle number is straightforward. Moreover, one can show that the above system of equations gives a stable homogeneous equilibrium in linear stability investigations, contrary to the theory of Eckart [14], therefore it can be considered as a minimal viable extension of the local equilibrium theory without the complexity of the Müller-Israel-Stewart one.

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